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MODELING SEPARATION BOUNDARIES IN THE FLOW OF A HEAVY FLUID OVER A WING PROFILE

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The presence of boundaries separating media has a substantial effect on the nature of the flow around a wing and the forces acting on it. In the plane case the separation boundary is a flow curve, where velocity undergoes a tangential jump when it passes through it, so a piecewise analytic function with an unknown curve of discontinuity in the region of the flow must be sought in order to construct a complex flow potential. If the [wing] contour is sufficiently far from the separation boundaries, modeling the separation lines by continuously distributed singularities within the framework of the theory of low-amplitude waves makes it possible to solve a wide circle of problems [1-4], in which the conditions on the contour are fulfilled exactly.

Here the method of distribution of singularities is used to solve problem in which a two-layered heavy fluid with a free surface flows around a profile located in the layer of the fluid. This problem is related to the "dead wave" phenomena [5], which is caused by the formation of waves on the boundary separating fluids of different density. We make note of [6], where an attempt was made to solve the problem by the method in [7]. However, the investigation in [6] was limited by the choice of an integral equation; only a contour of continuous curvature was studied; and the problem of defining the circulation was not treated.

1. In a system of coordinates bound to the [wing] profile C, we examine a steady-state flow of an ideal incompressible heavy fluid, which is limited by a free surface E_1 and consists of a layer of thickness H of density ρ_1 and an infinitely thick layer of density ρ_2 with a boundary E_2 separating the fluids. The Ox axis is directed against the flow; the Oy axis is vertical upwards, and the origin of the coordinates lies in the middle of the chord of C. The flow velocity at infinity ahead of the profile is parallel to the immobile separation boundary and is equal to $-V_j$ $(j = 1, 2)$.

In the representation of a potential flow, the problem reduces to determining the complex potentials of the perturbed flow $W_j^0(z)$ in the corresponding regions D_j, where D_1 represents

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the strip $h-H \leqslant y \leqslant h$, and D_2 represents the half plane $y \leqslant h-H$, excluding the region bounded by the contour C.

We denote $W_j^0(z)=V_jW_j(z)$, $m^-=\rho_1V_1^2/(\rho_1V_1^2+\rho_2V_2^2)$, $m^+=\rho_2V_2^2/(\rho_1V_1^2+\rho_2V_2^2)$, $m=m^+-m^-$, $\nu_1=g/V_1^2$, and $\nu_2=g(\rho_2-\rho_1)/(\rho_1V_1^2+\rho_2V_2^2)$. Then within the framework of the theory of low-amplitude waves, the boundary conditions of the problem for analytic functions $W_j(z)$ outside C are written in the form of no flow in the profile for $z \in C$

$$\operatorname{Im} W_2(z) = y + \psi_0 \; (\psi_0 = \text{const}); \tag{1.1}$$

the condition on the free surface

$$\operatorname{Re}[dW_1(z)/dz + i\nu_1 W_1(z)] = 0,\; y = h; \tag{1.2}$$

the dynamic and kinematic conditions on the boundary separating the fluids

$$\operatorname{Re}[m^+ dW_2(z)/dz - m^- dW_1(z)/dz + i\nu_2 W_2(z)] = 0,\; y = h - H; \tag{1.3}$$

$$\operatorname{Im}[W_2(z) - W_1(z)] = 0,\; y = h - H \tag{1.4}$$

and the condition at infinity

$$\lim_{x\to+\infty} |dW_j(z)/dz| = 0. \tag{1.5}$$

2. The method of solving the problem consists of a distribution of singularities with intensities $\mu_1(x + ih)$ and $\mu_2(x + ih - iH)$ along the unperturbed level of the free surface E_1 and the curve separating the fluids E_2. We initially examine the circulation-free flow around the profile C. The flow potential can be represented in the form

$$W_p(z) = W_{p\infty}(z) + v_1(z) + v_2(z) + \Phi_1(z) + \Phi_2(z), \tag{2.1}$$

where $W_{p\infty}(z)$ is the complex potential of the perturbed flow when an unbounded flow moves around the profile,

$$v_k(z)=\frac{1}{2\pi i}\int_{E_k}\frac{\mu_k(t)\,dt}{z-t},\quad \Phi_k(z)=\frac{1}{2\pi i}\int_{E_k}F_k(z,t)\,\mu_k(t)\,dt \qquad (k=1,2).$$

We assume that the integrals $v_k(z)$ exist.

In the corresponding region D_j the potential $W_p(z)$ will be given another index j, because the function $v_2(z)$ on the boundary E_2 in the approaches from D_1 and D_2 takes on different limiting values. The functions $F_k(z, t)$ are constructed in the parametric plane ζ based on the Milne-Thompson theorem of neighboring regions [8] such that condition (1.1) is satisfied by

$$F_k[f(\zeta), f(\tau)] = [\chi_k(\zeta,\tau) + G_k(\zeta,\tau)]/f'(\tau),\; G_k(\zeta,\tau) = 1/[\tau^2(\zeta - 1/\tau)],$$
$$\chi_k(\zeta,\tau) = [f(\zeta) - f(\tau) - f'(\tau)(\zeta-\tau)]/(\zeta-\tau)/[f(\zeta) - f(\tau)].$$

Here $z = f(\zeta)$ is a conformal mapping of the exterior the circle $C^0: |\zeta| = 1$ onto the exterior of the profile, so that $f(\infty) = \infty$ and $\zeta_a = -1$ corresponds to the trailing edge of the profile z_a, and $\tau = f^{-1}(t)$ for $t \in E_k$.

3. In order to determine $\mu_k(t)$ we use conditions (1.2)-(1.5). We transform the coordinates $z = z_1 + ih$. After we substitute (2.1) into (1.2) for $y_1 = 0$, we obtain

$$\operatorname{Re}\left\{\frac{dv_1(z_1)}{dz_1} + i\nu_1 v_1(z_1) + \left(\frac{d}{dz_1} + i\nu_1\right)[\Omega(z_1) + v_2(z_1)]\right\}_{z_1=x_1-i0} = 0 \tag{3.1}$$

where $\Omega(z_1) = W_{p\infty}(z_1) + \Phi_1(z_1) + \Phi_2(z_1)$. Condition (3.1) represents a singular integral equation relative to $\mu_1(t)$. It can be regularized by using the following approach. We note that (3.1) is equivalent to the equation

$$\operatorname{Re}\left\{\frac{dv_1(z_1)}{dz_1} + i\nu_1 v_1(z_1) + \left(\frac{d}{dz_1} - i\nu_1\right)[\bar\Omega(z_1) + \bar v_2(z_1)]\right\} = 0, \quad z_1 = x_1.$$

Because $v_1(z_1)$, $\bar v_2(z_1)$, and $\bar\Omega(z_1)$ are regular in the half plane $y_1 \leqslant 0$, then

$$\frac{dv_1}{dz_1} + i\nu_1 v_1 + \left(\frac{d}{dz_1} - i\nu_1\right)[\bar\Omega(z_1) + \bar v_2(z_1)] = i\nu_1 N \tag{3.2}$$

where N is a real constant. If we set $v_1(+\infty) = v_2(+\infty) = 0$, then from the condition (1.5) it follows that N = 0. By solving (3.2) relative to $v_1(z_1)$ and finding the real part of the limit for $z_1 \to x_1$ (the limiting transition in the singular limit is accomplished through the Sokhotskii formula) in terms of ζ for $z_1 = f(\zeta) - ih$, we obtain

$$\mu_1(\zeta_1) = \varphi_1(\zeta_1) + \operatorname{Im}\int_{T_1} L_1(\zeta_1, \tau)\mu_1(\tau)\,d\tau + \operatorname{Im}\int_{T_2} L_4(\zeta_1, \tau)\mu_2(\tau)\,d\tau,$$

$$\varphi_1(\zeta_1) = 2\operatorname{Re}\left\{W_{p\infty}(\zeta_1) - 2\exp[iv_1 f(\zeta_1)]\int_\infty^{\zeta_1}\exp[-iv_1 f(v)]\,W'_{p\infty}(v)\,dv\right\},$$

$$W_{p\infty}(\zeta_1) = -(K\zeta_1 + \overline{K}/\zeta_1) + f(\zeta_1), \quad K = f'_\zeta(\infty), \tag{3.3}$$

$$L_1(\zeta_1, \tau) = \frac{1}{\pi}\left\{\chi_1(\zeta_1, \tau) + G_1(\zeta_1, \tau) - 2\exp[iv_1 f(\zeta_1)]\int_\infty^{\zeta_1}\exp[-iv_1 f(v)]\left[G'_{1v}(v, \tau) + \chi'_{1v}(v, \tau)\right]dv\right\},$$

$$L_4(\zeta_1, \tau) = \frac{1}{\pi}\left\{1/(\zeta_1 - \tau) + G_2(\zeta_1, \tau) - 2\exp[iv_1 f(\zeta_1)]\int_\infty^{\zeta_1}\exp[-iv_1 f(v)]\left[-1/(v-\tau)^2 + G'_{2v}(v, \tau)\right]dv\right\},$$

where T_1: $\zeta_1 = f^{-1}(x + ih)$ and T_2: $\zeta_2 = f^{-1}(x + ih - iH)$ are images of the unperturbed boundaries separating the fluids in the parametric plane.

In order to obtain the second equation for $\mu_k(t)$ we examine the conditions on the curve separating the fluids where they intersect the abscissa, that is, we transform the coordinates $z = z_2 + ih - iH$. The potential in the form (2.1) satisfies the condition (1.4). After we conduct the sequence of operations (3.1) and (3.2) on (1.3), in which $v_1(z_2)$ is not subjected to the joining operation, we find

$$\mu_2(\zeta_2) = \varphi_2(\zeta_2) + \operatorname{Im}\int_{T_2} L_2(\zeta_2, \tau)\mu_2(\tau)\,d\tau + \operatorname{Im}\int_{T_1} L_3(\zeta_2, \tau)\mu_1(\tau)\,d\tau,$$

$$\varphi_2(\zeta_2) = -2\operatorname{Re}\left\{W_{p\infty}(\zeta_2) - 2m^+\exp[iv_2 f(\zeta_2)]\int_\infty^{\zeta_2}\exp[-iv_2 f(v)]\,W'_{p\infty}(v)\,dv\right\},$$

$$L_2(\zeta_2, \tau) = \frac{1}{\pi}\left\{G_2(\zeta_2, \tau) + \chi_2(\zeta_2, \tau) - \right.$$

$$\left. - 2m^+\exp[iv_2 f(\zeta_2)]\int_\infty^{\zeta_2}\exp[-iv_2 f(v)]\left[G'_{2v}(v, \tau) + \chi'_{2v}(v, \tau)\right]dv\right\}, \tag{3.4}$$

$$L_3(\zeta_2, \tau) = \frac{1}{\pi}\left\{f'(\tau)/[f(\zeta_2) - \bar{f}(\tau) - 2i(h - H)] + G_1(\zeta_2, \tau) + \chi_1(\zeta_2, \tau) - \right.$$

$$- 2m^+\exp[iv_2 f(\zeta_2)]\int_\infty^{\zeta_2}\exp[-iv_2 f(v)]\left[G'_{1v}(v, \tau) + \chi'_{1v}(v, \tau)\right]dv +$$

$$\left. + 2m^-\exp[iv_2 f(\zeta_2)]\int_\infty^{\zeta_2}\exp[-iv_2 f(v)]\frac{f'(\tau)f'(v)}{[f(v) - \bar{f}(\tau) - 2i(h-H)]^2}\,dv\right\}.$$

4. In the case of circulational flow around the profile C, the complex potential will be sought in the form

$$W_j(z) = W_{pj}(z) + \Gamma_1 W_{\Gamma j}(z). \tag{4.1}$$

Here $\Gamma_1 = \Gamma/V_2$; Γ is the circulation value; $W_{\Gamma j}(z)$ is the piecewise analytic function outside C which satisfies conditions (1.2)-(1.5) and the conditions

$$\operatorname{Im} W_{\Gamma 2}(z) = \psi_1,\ z \in C\ (\psi_1 = \text{const}),\ \Delta_c W_{\Gamma 2}(z) = 1, \tag{4.2}$$

where Δ_c is the increment of the function in going around the contour C. The function $W_{\Gamma j}(z)$ is sought in the form analogous to (2.1), where $\mu_k(t)$ must be replaced by $\mu_{\Gamma k}(t)$ and $W_{p\infty}(z)$ is replaced by $W_{\Gamma\infty}(z)$. We examine the expression

$$\frac{1}{2\pi i}\ln\frac{\gamma(z)}{z - z_\Gamma} = \frac{1}{2\pi i}\ln\frac{\zeta - 1/\bar{\zeta}_\Gamma}{\zeta - \zeta_\Gamma}$$

where $(z_\Gamma = f(\zeta_\Gamma) \notin D_1 \cup D_2$ is an arbitrarily chosen point. We can note that the function

$$W_{\Gamma\infty}(z)=\frac{1}{2\pi i}\ln\frac{\gamma(z)}{z-z_\Gamma},\quad z\in D_1\cup D_2$$

satisfies conditions (1.4) and (4.2); therefore the potential $W_{\Gamma j}(z)$ also satisfies these conditions.

The system of integral equations for $\mu_{\Gamma k}(t)$ is also obtained from conditions (1.2) and (1.3) and is reduced to the form (3.3) and (3.4) with the same kernel L_n ($n = 1,\ldots,4$) and with the following free parts

$$\varphi_{\Gamma 1}(\zeta_1)=\frac{1}{\pi}\operatorname{Im}\left\{\ln\frac{(\zeta_1-1/\bar\zeta_\Gamma)[f(\zeta_1)-f(\zeta_\Gamma)]}{(\zeta_1-\zeta_\Gamma)[f(\zeta_1)-\bar f(\zeta_\Gamma)-2ih]}-\right.$$

$$\left.-2\exp[i\nu_1 f(\zeta_1)]\int\limits_\infty^{\zeta_1}\exp[-i\nu_1 f(v)][1/(v-1/\bar\zeta_\Gamma)+f'(v)/[f(v)-f(\zeta_\Gamma)]-1/(v-\zeta_\Gamma)]\,dv\right\},$$

$$\varphi_{\Gamma 2}(\zeta_2)=\frac{1}{\pi}\operatorname{Im}\left\{\ln\frac{(\zeta_2-1/\bar\zeta_\Gamma)[f(\zeta_2)-f(\zeta_\Gamma)]}{(\zeta_2-\zeta_\Gamma)[f(\zeta_2)-\bar f(\zeta_\Gamma)-2i(h-H)]}-\right.$$

$$-2m^+\exp[i\nu_2 f(\zeta_2)]\int\limits_\infty^{\zeta_2}\exp[-i\nu_2 f(v)][1/(v-1/\bar\zeta_\Gamma)+f'(v)/[f(v)-f(\zeta_\Gamma)]$$

$$\left.-1/(v-\zeta_\Gamma)]\,dv+2m^-\exp[i\nu_2 f(\zeta_2)]\int\limits_\infty^{\zeta_2}\frac{f'(v)\exp[-i\nu_2 f(v)]}{f(v)-\bar f(\zeta_\Gamma)-2i(h-H)}\,dv\right\}.$$

In deriving equations for $\mu_{\Gamma k}(t)$, the terms containing $\ln(z - z_\Gamma)$ do not take part in the joining.

The value of the circulation Γ_1 is found from the Zhukovskii–Chapling postulate on the finiteness of the velocity at the sharp edge of the profile:

$$\Gamma_1=\frac{2\pi\operatorname{Im}\left(-K+\bar K/\zeta_a^2\right)+\operatorname{Re}[J_1(\zeta_a)+J_2(\zeta_a)]}{\operatorname{Re}[1/(\zeta_a-1/\bar\zeta_\Gamma)-1/(\zeta_a-\zeta_\Gamma)-J_{\Gamma 1}(\zeta_a)-J_{\Gamma 2}(\zeta_a)]},$$

$$J_k(\zeta_a)=\int\limits_{T_k}I(\zeta_a,\tau)\,\mu_k(\tau)\,d\tau,\quad J_{\Gamma k}(\zeta_a)=\int\limits_{T_k}I(\zeta_a,\tau)\,\mu_{\Gamma k}(\tau)\,d\tau,$$

$$I(\zeta_a,\tau)=1/(\zeta_a-\tau)^2+1/[\bar\tau(\zeta_a-1/\bar\tau)]^2.$$

5. The form of the free surface and the curve separating the fluids is sought on the basis of the relationships [9]

$$\delta_1(x_1)=\frac{1}{\nu_1}\operatorname{Re}\left[\frac{dW_1(z_1)}{dz_1}\right]\quad\text{for}\quad y_1=0;\tag{5.1}$$

$$\delta_2(x_2)=\frac{1}{\nu_2}\operatorname{Re}\left[m^+\frac{dW_2(z_2)}{dz_2}-m^-\frac{dW_1(z_2)}{dz_2}\right]\quad\text{for}\quad y_2=0\tag{5.2}$$

where $\delta_k(x_k)$ is the elevation of the boundary separating the media above the upperturbed level.

In view of the presence of singular integrals in Eqs. (5.1) and (5.2), the terms containing them are transformed on the basis of the properties of derivatives of Cauchy-type integrals [10]. Finally we have

$$\delta_1(\zeta_1)=\frac{1}{\nu_1}\operatorname{Re}\left\{\Omega_1(\zeta)+\frac{1}{2\pi i f'(\zeta)}\int\limits_{T_2}[-1/(\zeta-\tau)^2+G'_{2\zeta}(\zeta,\tau)]\,\mu_2^0(\tau)\,d\tau\right\}_{\zeta=\zeta_1}+\mu_1^{0\prime}(\zeta_1)/[2\nu_1 f'(\zeta_1)],\quad\delta_2(\zeta_2)=$$

$$=\frac{m}{\nu_2}\left\{\Omega_2(\zeta)+\frac{1}{2\pi i f'(\zeta)}\int\limits_{T_1}[-1/(\zeta-\tau)^2+G'_{1\zeta}(\zeta,\tau)]\,\mu_1^0(\tau)\,d\tau\right\}_{\zeta=\zeta_2}+\mu_2^{0\prime}(\zeta_2)/[2\nu_2 f'(\zeta_2)],\tag{5.3}$$

$$\Omega_k(\zeta)=\frac{1}{f'(\zeta)}\left\{W'_{p\infty}(\zeta)+\Gamma_1 W'_{\Gamma\infty}(\zeta)+\frac{1}{2\pi i}\int\limits_{T_k}[\chi'_{k\zeta}(\zeta,\tau)+G'_{k\zeta}(\zeta,\tau)]\,d\tau\right\},$$

$$\mu_k^0(\tau)=\mu_k(\tau)+\Gamma_1\mu_{\Gamma k}(\tau).$$

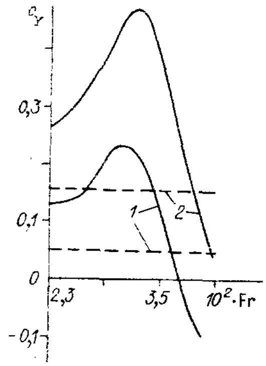

Fig. 1

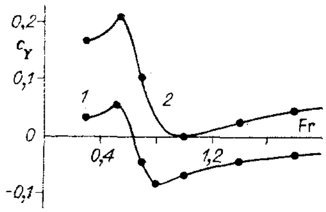

Fig. 2

6. In order to solve the system of Fredholm equations of the second kind – (3.3) and (3.4) – we use the method of successive approximations. For the zero-order approximation we can take the solution of Eq. (3.3) for $\mu_2(\tau) = 0$, which gives the solution for flow around the contour under a free surface of a homogeneous fluid. The zero-order approximation can also serve as the solution of Eq. (3.4) for $\mu_1(\tau) = 0$, which corresponds to the solution of the problem of the motion of the profile under the surface separating two fluids of different density. The values of μ_1 obtained from solving (3.3) are substituted into Eq. (3.4) and the values μ_2 from (3.4) are substituted into (3.3). The process is repeated until a specified order of accuracy is obtained. In an analogous manner, the system is solved for $\mu_{\Gamma k}$. After we calculate the circulation value from Eq. (4.4), we can determine the complex potential and then use the Chapling formula [6] to find

$$X - iY = \frac{i\rho_2}{2} \oint_{C^0} \left[\frac{dW_2^0(\zeta)}{d\zeta}\right]^2 \frac{d\zeta}{f'(\zeta)} - i\rho_2 V_2 \Gamma$$

to find the lifting force Y and the wave resistance X. From the formula

$$c_p = 1 - \left|\frac{1}{f'(\zeta)} \frac{dW_2^0(\zeta)}{d\zeta}\right|^2 \Big/ V_2^2$$

we determine the pressure coefficient on the profile. The form of the free surface and the curve separating the fluids is calculated from (5.3), where the values of the derivatives of the singularity densities are found by differentiating Eqs. (3.3) and (3.4).

The conformal mapping of the exterior of the circle onto the exterior of the profile was done by the method of [11]. The calculations were done on a PC AT. In the particular case of flow of a heavy fluid with one separating boundary, we obtained the published data [2, 4, 9, 12] for a circular cylinder and a wing profile.

Figures 1-4 show the calculated results for the NACA profile 66_1-012. Figures 1 and 2 give the calculated results of the lift coefficient $c_Y = 2Y/(\rho_2 V_2^2 L)$ as a function of the Froude number $Fr = V_2/\sqrt{gL}$ for the following flow parameters: h/L = 0.8, H/L = 0.4, $\rho_2/\rho_1 = 1.01$, $V_2 = V_1$, and $\alpha = 0$ and 1° (curves 1 and 2), where L = 1 is the length of the chord of the profile and α is the attack angle. The separation of the interval of small Fr (Fig. 1) ($Fr < Fr^*$) is related to the fact that it is namely for small Fr (for given values of H/L, ρ_2/ρ_1, and $Fr^* = 0.0632$) that periodic waves with amplitudes significantly above those on a free surface exist behind the profile on the fluid separation boundary. We note that doing calculations with sufficient accuracy near the critical Fr^* requires a large amount of time In this regard, the calculated results are done with some standoff from Fr^*. Examples of the calculation of the internal waves for Fr = 0.032 and 0.038 (curves 1 and 2) for $\alpha = 0$ are shown in Fig. 3. The free surface for these Fr values remain unperturbed (the "dead wave" phenomena). The pressure distribution along the profile as a function of the distance x^*/L, which is calculated starting from the nose along the chord of the profile, is shown in Fig. 4 for the same parameters. The dashed lines and the points in Figs. 1 and 2 represent the characteristics for the problem of flow around the profile under a free surface of a homogeneous fluid ($\rho_1 = \rho_2$). As can be seen in Fig. 2 for $Fr > Fr^*$, the effect of the fluid separation boundary remains insignificant in comparison to the effect of the free surface.

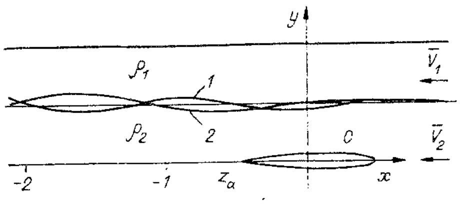

Fig. 3

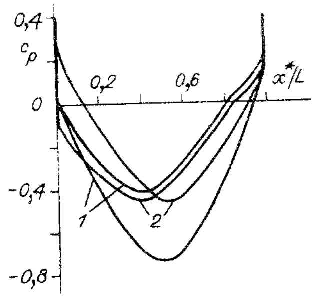

Fig. 4

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